Descartes' Analytic Geometry:

Plotting the Future of Mathematics

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Paper

Descartes' Analytic Geometry: Plotting the Future of Mathematics

René Descartes dipped his quill pen into the ink. "This is one thing which I believe the ancient mathematicians did not observe," he wrote, "for otherwise they would not have put so much labor into writing so many books in which the very sequence of the propositions shows that they did not have a sure method of finding all, but rather gathered together those propositions on which they had happened by accident."[1] What is the "thing" that the early mathematicians had missed? It was analytic geometry, a method invented by Descartes that used algebraic equations to describe geometric objects. Descartes discovered that he could solve difficult mathematical problems using this method. The key idea was to use coordinate axes to bring algebra and geometry together. This was an innovation that enabled mathematicians to solve complex problems and invent new forms of mathematics such as calculus, which laid the foundation for modern science and technology.

The history of mathematics begins with geometry. Euclid, a Greek mathematician, lived around 300 BC in Alexandria, Egypt. He invented a form of geometry that we still use today. When he wrote his book, *The Elements,* he started with only several axioms, or givens. For example, he stated that there is only one line connecting two points, and that all right angles are equal.[2] Then he built on these postulates, proving more and more theorems. But Euclid never used equations, and his

[1] René Descartes, *The Geometry,* Vol. 31 of *Great Books of the Western World,* ed. Robert Hutchins, (Chicago: Britannica, 1989), 298.
[2] Euclid, *The Thirteen Books of Euclid's Elements*, Vol. 11 of *Great Books of the Western World,* ed. Robert Hutchins, (Chicago: Britannica, 1989), 2.

geometric diagrams eventually became too complex and confusing as shown in Appendix 1.

Apollonius was another Greek mathematician. Born in Perga around 262 BC, he worked in Egypt. He wrote several books, including one called *Conics.*[3] A conic, or conic section, results when one cuts a hollow double cone with a plane. Conic sections can be parabolas, hyperbolas, ellipses, or circles.[4] Apollonius graphed conic sections, and he also drew lines that looked like coordinate axes. But his axes were based on what he was graphing and were not defined before the problem. Therefore, his axes were never in the same position and are not considered true coordinate axes.[5] Nevertheless, it was an important first step toward analytic geometry.

The Persians discovered and studied what they called Al-Jabr, which means "restoration" or "completion".[6] Today we simplify it to the word "algebra." Al-Khwarizmi, one of the earliest Persian mathematicians, was born around 780. He discovered how to do basic algebra. For example, he could simplify equations and solve simple quadratic equations. Al-Khwarizmi regarded algebra and geometry as separate fields and therefore did not attempt to combine them.[7] About 300 years later Omar Khayyam, another Persian, wrote several books on algebra, one of which mentioned the difficulties with Euclid's geometry.[8]

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[3] Apollonius, *Conics I,* Vol. 11 of *Great Books of the Western World,* ed. Robert Hutchins, (Chicago: Britannica, 1989), 603.
[4] Eugene D. Nichols et al., *Holt Algebra 2 with Trigonometry*, (New York: Holt, Rinehart and Winston, 1986), 324.
[5] Carl B. Boyer, *History of Analytic Geometry*, (Dover Publications Minneola, New York. 2004), 46.
[6] Jeremy Gray and Karen Hunger Parshal, *Episodes in the History of Modern Algebra (1800-1950),* (Providence, RI: American Mathematics Society, 2007), 3.
[7] Ibid.
[8] Boyer, *History of Analytic Geometry*, 43.

Nicole Oresme, a medieval mathematician, was born in 1323 in Normandy, France. He made another important step towards analytic geometry. He plotted curves with the use of fixed axes, and he also plotted points, but he did not use algebraic equations to describe his curves.[9]

Analytic geometry would have to wait two more centuries before the time was ripe for its full development. It would have to wait for someone talented enough to bring algebra and geometry together in the proper way. No one seemed to possess the creativity to accomplish this.

Except for René Descartes.

Descartes was born to middle-class parents on March 31, 1596 in La Haye, France.[10] His mother died of tuberculosis when he was only one. He was a sickly child, suffering from "a dry cough and a pale color," which he believed he inherited from her.[11]

When Descartes turned eleven, he entered a Jesuit college in La Flèche. His teachers would let him sleep until noon because of his poor health, but while in bed, he developed new ideas.[12] He believed that by reasoning, he could overcome his troubles. "I think this inclination caused [my illness]," he later wrote, "gradually to disappear."[13]

After he graduated in 1615, he studied law at the University of Poitiers as his father wished. He earned a license in law in 1616,[14] but he quit law after a few months to

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[9] Boyer, *History of Analytic Geometry*, 45-47.
[10] Kurt Smith, "Descartes' Life and Works", *The Stanford Encyclopedia of Philosophy (Fall 2008 Edition)*, ed. Edward N. Zalta, available from http://plato.stanford.edu/archives/fall2008/entries/descartes-works, accessed 19 February 2010.
[11] Stephen Gaukroger, *Descartes: An Intellectual Biography*, (New York: Oxford University Press, 1995), 16-20.
[12] Chikara Sasaki, *Descartes' Mathematical Thought*, Volume 237 of Boston Studies in the Philosophy of Science, (New York: Springer Science+Business Media, 2003), 45-47.
[13] Gaukroger, *Descartes: An Intellectual Biography*, 16.
[14] Forrest E. Baird and Walter Kaufmann, *From Plato to Derrida*, (New Jersey: Pearson Prentice Hall, 2008), 373–377.

study philosophy. He loved to take walks in the forest, thinking about new ideas, and his mind was always wandering from one topic to another. He was a restless philosopher and moved frequently over the next several years, living in Ulm, Paris, Amsterdam, Utrecht, Leiden, and Santpoort.[15] He even traveled to Denmark[16] and Italy.[17]

In the summer of 1618, Descartes entered the Dutch army under Maurice of Nassau, the protestant ruler of the Dutch Republic. He joined an engineer corps, helping to design and build siege weapons.[18] At this time there was a temporary peace in Holland's war of independence from Spain.[19]

He met a mathematician named Isaac Beeckman while marching through Breda, a town in the southern part of the Netherlands. Beeckman was interested in Descartes' ideas. They talked frequently about mathematics and the dead ends that mathematicians had encountered. Beeckman was like a teacher to him, and he kindled Descartes' interest in mathematics.[20]

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[15] Smith, *"*Descartes' Life and Works*", The Stanford Encyclopedia of Philosophy.*

[16] René Descartes, Letter to Isaac Beeckman, 29 April 1619, in *The Philosophical Writings of Descartes.* Volume 3*: The Correspondence,* ed. John Cottingham, Robert Soothoff, Dugald Murdoch, and Anthony Kenny, (New York: Cambridge University Press, 1991), 5.

[17] Richard Watson, *Cogito, Ergo Sum: The Life of Rene Descartes*, (Boston: David R. Godine, 2007), 119-120.

[18] Ibid.

[19] Donald Kagan, Steven Ozment, and Frank M. Turner, *The Western Heritage*, (Upper Saddle River, New Jersey: Prentice Hall, 1998), 432.

[20] Smith, *"*Descartes' Life and Works*", The Stanford Encyclopedia of Philosophy*; René Descartes, Letter to Isaac Beeckman, 23 April 1619, in *The Philosophical Writings of Descartes.* Volume 3*: The Correspondence,* ed. John Cottingham, Robert Soothoff, Dugald Murdoch, and Anthony Kenny, (New York: Cambridge University Press, 1991), 4. Descartes wrote to Beeckman, "For it was you alone who roused me from my state of indolence, and reawakened the learning which by then had almost disappeared from my memory; and when my mind strayed from serious pursuits, it was you who led it back to worthier things."

When Descartes left the army, he focused his attention on the possibility of a form of mathematics that would combine algebra and geometry. He wrote many letters to Beeckman, describing the things that he had discovered and asking for advice.[21]

In one letter, he described a new method for determining one's position from the stars.[22] Navigators already knew how to determine latitude, but Descartes solved the more difficult problem of determining longitude. In another letter, he told Beeckman that he had started writing his book *Geometry*, and he thanked him for encouraging his interest in mathematics.[23]

There is a long-standing myth that describes how Descartes discovered analytic geometry. He was lying in bed when he saw a fly crawling on the ceiling.[24] Being a creative mathematician, he wondered if he could plot the fly's path on a geometric plane. Grabbing a sheet of paper, he drew a graph of the ceiling. He drew a horizontal line called the *x* axis and a vertical line called the *y* axis. He marked numbers on these axes like those of a ruler. Then he took the point where the axes met and called it the origin. With growing excitement he watched the fly land on point after point and realized that every point could be described with a pair of numbers: its distance from the *x* axis and its distance from the *y* axis.

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[21] René Descartes, Letters to Isaac Beeckman, in *The Philosophical Writings of Descartes.* Volume 3*: The Correspondence,* ed. John Cottingham, Robert Soothoff, Dugald Murdoch, and Anthony Kenny, (New York: Cambridge University Press, 1991), 1-4, 26-28, 45-47. Although this book contains only six letters to Beeckman, it is clear from Descartes' statements that they maintained a frequent correspondence.

[22] René Descartes, Letter to Isaac Beeckman, 26 March 1619, in *The Philosophical Writings of Descartes.* Volume 3*: The Correspondence,* ed. John Cottingham, Robert Soothoff, Dugald Murdoch, and Anthony Kenny, (New York: Cambridge University Press, 1991), 3.

[23] Descartes, Letter to Isaac Beeckman, 23 April 1619.

[24] Devlin, Keith J., *The Language of Mathematics: Making the Invisible Visible*, (New York: Macmillan 2000), 156.; Reimer, Luetta and Wilbert Reimer, *Mathematicians are People, Too*, (Palo Alto, California: Dale Seymour Publications, 1995), 39.; K. M. Vickers and M. J. Tipler, *New National Curriculum Mathematics: Without Answers,* New National Curriculum Mathematics Series, (Cheltenham, UK: Nelson Thornes, 1996), 133.

Although there is no evidence that this story is true,[25] it is a good illustration of what Descartes discovered. In his book *Geometry*, Descartes shows a curve that he needs to analyze algebraically. He does not use the term "coordinate axes", but his diagram contains them, as shown in Appendix 3. He writes, "I choose a straight line, as AB, to attach to refer all its points…" This is the vertical y axis. He continues, "…and in AB I choose a point A at which to begin the investigation."[26] This is the origin. Descartes' diagram also includes a horizontal line GA, which is the x axis.

So far, Descartes has drawn the x and y coordinate axes and identified the origin. But he has not yet assigned the x and y coordinates. He continues: "Then I draw through C the line CB parallel to GA [the x axis]. Since CB and BA are unknown and indeterminate quantities, I shall call one of them y and the other x."[27] Voilà! With this statement, Descartes has described x and y coordinates.

Descartes continues by setting up the algebraic equation that determines the curve and then uses it to identify the curve: "Therefore the required equation is

$$y^2 = cy - \frac{cx}{b}y + ay - ac\,.$$

From this equation we see that the curve EC [is] in fact, a hyperbola."[28]

This was Descartes' key idea for analytic geometry: given a geometric problem, he constructed the x and y coordinate axes, determined the algebraic equation, and used it to solve the geometric problem. To modern mathematicians, it seems obvious because

[25] The author has found no evidence of this story in any primary source, including Descartes' books and letters. However, it appears in many secondary sources without reference.
[26] Descartes, *The Geometry,* 306.
[27] Ibid, 307.
[28] Ibid, 307. The author is indebted to Dr. Mark D. Pritt for his help in understanding Descartes' idea.

they learn mathematics this way as students.[29] To Descartes, it was a new approach that avoided the "obscurity and embarrassment"[30] that plagued older methods of doing mathematics. One can almost feel his excitement as he writes, "But I shall not stop to explain this in more detail, because I should deprive you of the pleasure of mastering it yourself…"[31]

Descartes published his mathematical innovation in 1637. Originally published in French as *La Géométrie*, it was quickly translated into Latin, the language of mathematicians at the time. Interestingly, it was not even an entire book. It was only an appendix to his book *Discourse on Method,* in which he wrote his famous statement, "Cogito, ergo sum," which means, "I think, therefore I am."[32]

Descartes was a master at combining different ideas to create new concepts. To create analytic geometry, he combined two very different things – algebra and geometry. He showed that one could begin with truths about algebra and deduce truths about geometry, and vice versa. Likewise the statement "I think, therefore I am" combines two very different things – mind and being. Descartes showed that one could begin with truths about the mind and deduce truths about being. For example, the fact that I am thinking proves that I exist. Descartes also showed that truths about the mind could be deduced from truths about being. For example, "…all the things that we very clearly and very distinctly conceive of are true, is certain only because God is or exists, and that He is a Perfect Being, and that all that is in us issues from Him."[33]

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[29] Interview with Dr. Mark D. Pritt, Systems Engineer, Lockheed Martin Corp., personal interview with the author, 17 February 2010.
[30] Descartes, *The Geometry,* 300.
[31] Ibid, 297.
[32] René Descartes, *Discourse on Method,* Vol. 31 of *Great Books of the Western World,* ed. Robert Hutchins, (Chicago: Britannica, 1952), 51.
[33] Ibid, 53.

Descartes' ideas spread quickly throughout Europe. Pierre de Fermat, a French lawyer and amateur mathematician, read Descartes' *Geometry* and stated that Descartes "…has made so much headway and has taken such a difficult path for these tangents in his Geometry."[34]

But Descartes was also criticized. When his friend Beeckman accused him of plagiarism, Descartes responded by writing, "You reproach me, without any reason or basis," and then he went on to insult him, "…for it occasionally happens, that even when the most incompetent person discusses philosophy, he says many things which by sheer chance coincide with the truth."[35] The philosopher and mathematician Blaise Pascal criticized Descartes for his religious views. In his book *Pensées*, he wrote, "I cannot forgive Descartes. In all his philosophy he would have been quite willing to dispense with God... he has no further need of God."[36]

When the Inquisition condemned Galileo in 1633, Descartes feared that the criticism his own ideas received could develop into persecution. He had been working on a physics book called *The World* in which, like Galileo, he stated that the earth revolved about the sun.[37] Galileo was condemned for this heliocentric belief, and Descartes feared that he too might be condemned.[38] As a result, he decided not to publish the book.

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[34] Letter from Fermat to Mersenne, February 1638, available from http://wlym.com/~animations/fermat/16380200%20Fermat%20to%20Mersenne.pdf, accessed 20 February 2010.

[35] René Descartes, Letter to Isaac Beeckman, 17 October 1630, in *The Philosophical Writings of Descartes.* Volume 3*: The Correspondence,* ed. John Cottingham, Robert Soothoff, Dugald Murdoch, and Anthony Kenny, (New York: Cambridge University Press, 1991), 26-27. In this letter, Descartes tries to persuade Isaac Beeckman that he did not plagiarize his ideas.

[36] Blaise Pascal, *Pensées*, translated by W. F. Trotter, (Charleston, SC: BiblioBazaar, LLC, 2009), 33.

[37] René Descartes, *The World or Treatise on Light*, translated by Michael S. Mahoney, available from http://www.princeton.edu/~hos/mike/texts/descartes/world/worldfr.htm, accessed 12 May 2010.

[38] René Descartes, Letter to Mersenne, November 1633, in *The Philosophical Writings of Descartes.* Volume 3*: The Correspondence,* ed. John Cottingham, Robert Soothoff, Dugald Murdoch, and Anthony Kenny, (New York: Cambridge University Press, 1991), 40-41. Descartes writes to his friend Father Mersenne, "In fact I had intended to send you my *World* as a New Year gift [but] I took the trouble to

Fortunately, this fate did not befall *Geometry.* Half a century later it enabled Isaac Newton to discover calculus, the laws of motion, and gravitational theory. Before Newton could make these discoveries, he had to learn mathematics. His nephew, John Conduitt, recounts that Newton bought a book about astrology, which is the attempt to predict the future from the stars. Newton did not understand the mathematics in this book, so he purchased a copy of Euclid's *Elements.* It did not answer his questions, so he bought a copy of *Geometry.* After reading it, "[Newton] laid Euclid aside as a trifling book, & was soon convinced of the vanity & emptiness of the pretended science of Iudicial astrology."[39] Interestingly, Newton later changed his mind about Descartes' *Geometry* and criticized it as "the Analysis of the Bunglers in Mathematicks."[40] Despite his harsh criticism, Newton built his theories on Descartes' ideas.

Descartes' ideas also influenced Leibniz, a German mathematician who also invented calculus. He wrote in a letter to a friend, "…I confess that the reading of Descartes is very useful and very instructive, and I like incomparably more to have to do with a Cartesian than with a man from some other school."[41]

When Newton and Leibniz invented calculus, they built on Descartes' ideas. Without analytic geometry, calculus and modern mathematics would be impossible.

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inquire in Leiden and Amsterdam whether Galileo's *World System* was available, for I thought I had heard that it was published in Italy last year. I was told that it had indeed been published but that all the copies had immediately been burnt at Rome, and that Galileo had been convicted and fined. I was so astonished at this that I almost decided to burn all my papers or at least to let no one see them." Descartes concludes, "But for all the world I did not want to publish a discourse in which a single word could be found that the Church would have disapproved of; so I preferred to suppress it rather than to publish it in a mutilated form."

[39] Conduitt's account of Newton's life at Cambridge. Keynes Ms. 130.4, King's College Library, Cambridge. Available from http://www.newtonproject.sussex.ac.uk/view/texts/normalized/THEM00167, accessed 15 February 2010.

[40] Richard S. Westfall, *Never at Rest: A Biography of Isaac Newton*, (New York: Cambridge University Press, 1983), 380.

[41] Gottfried Wilhelm Leibnez, letter to Philipp, 1679, in *The Philosophical Works of Leibniz*, ed. George Martin Duncan, (New Haven: Tuttle, Morehouse & Taylor, 1908), 2.

Practically all of modern technology depends on Descartes' Cartesian coordinate system.[42]

Analytic geometry lays the groundwork for calculus, trigonometry, and other forms of mathematics. While it might be possible to work complex problems without analytic geometry, it would be much harder and much more complicated. Analytic geometry simplifies mathematics immensely.[43] It allows mathematicians to work with structures that are in higher dimensions than humans can visualize.[44] It is essential to differential geometry, which uses ideas in calculus to solve geometric problems,[45] and topology, which is a form of geometry.[46] Since 1985, scientists have used analytic geometry to solve problems in cryptography, the writing of messages in secret code.[47]

Analytic geometry is vital to modern science and engineering. Physicists depend on it when working with vectors and calculus.[48] Astronomers use it to control the pointing of telescopes with trigonometry.[49] Biologists use analytic geometry in spectroscopy, and it enables them to solve equations resulting from biological theories.[50]

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[42] Eugene D. Nichols et al., *Holt Algebra 1*, (New York: Holt, Rinehart and Winston, 1986), 292.
[43] Interview with Mr. John Nicodemus, teacher at Frederick Community College, email interview, 29 March 2010.
[44] Dr. Mark D. Pritt, personal interview with the author, 17 February 2010.
[45] Interview with Dr. Chikako Mese, Chair and Professor, Department of Mathematics, Johns Hopkins University, email interview, 31 March 2010.
[46] Interview with Dr. Richard Brown, Director of Undergraduate Studies, Department of Mathematics, Johns Hopkins University, email interview, 31 March 2010. Dr. Brown stated, "My field of research would not exist without analytic geometry."
[47] Interview with Dr. Lawrence Washington, Professor of Mathematics, University of Maryland, email interview, 30 March 2010. Dr Washington wrote, "Analytic geometry was not important in number theory until around 1900, and it was not important in cryptography until around 1985. In both these fields, once analytic geometry started being used, people saw how useful it was and continued to make great use of it."
[48] Douglas C. Giancoli, *Physics for Scientists & Engineers, 4th ed.,* (New Jersey: Pearson Prentice Hall, 2008), chs. 3, 7.
[49] Interview with Dr. Christopher Stubbs, Professor of Physics and Astronomy, Harvard University, email interview, 1 April 2010.
[50] Interview with Dr. Blake Hill, Associate Professor of Biology and Chemistry, Johns Hopkins University, email interview, 31 March 2010.

Cartographers use it to create maps.[51] Navigators use it to graph latitude and longitude. The Global Positioning System (GPS) that many people use today for navigation would not be possible without analytic geometry.[52] Civil engineers use algebraic equations to determine how much weight a bridge can support and then use geometric diagrams to build the bridge. Project engineers use analytic geometry for stormwater management, floodplain analysis, and road design for commercial buildings and housing developments.[53] Aerospace engineers use analytic geometry to control satellites and spacecraft.[54]

Analytic geometry is also an important part of computer programming.[55] A computer mouse senses movement in *x* and *y* coordinates, and the computer uses algebraic equations to move the cursor and control objects on the screen. Video games use analytic geometry to control objects in an imaginary world, which is defined by a Cartesian coordinate system. According to a computer programming text, "All of the objects in a game are moved using an algorithm that calculates the *x* and *y* velocity values, which are used to update the object's position on the screen."[56] Analytic geometry is used in image processing for photography and reconnaissance, where *x* and *y* coordinates are used to control image pixels.[57] It is also used in text processing, where documents are represented by the number of times each word appears. These numbers are graphed in

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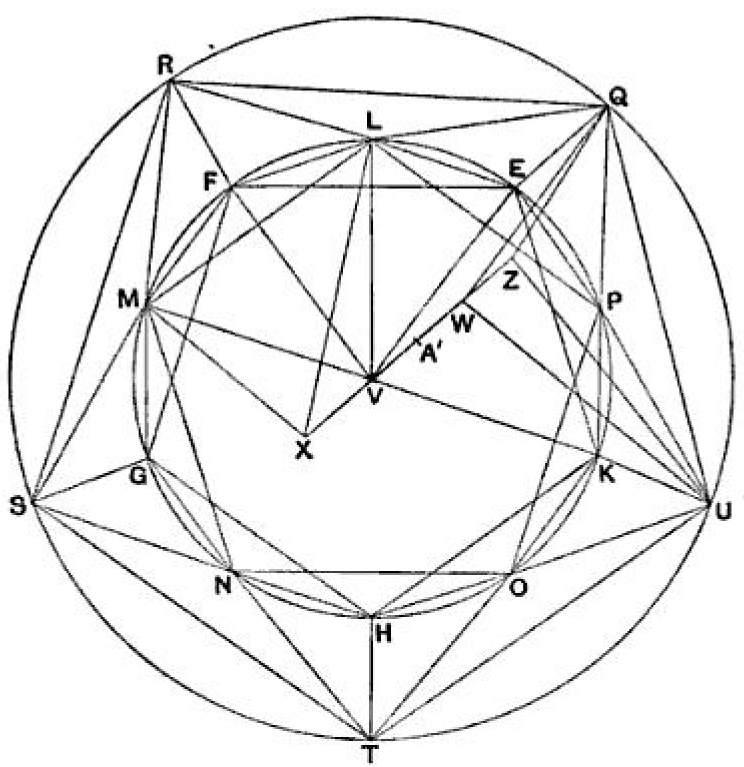
[51] John C. Hudson, ed., *Goode's World Atlas 20th Edition,* (Rand McNally, 2000), vi-xi.
[52] National Space-Based Positioning, Navigation, and Timing Coordination Office, "Global Positioning System: Serving the World", available from < http://www.gps.gov/>, accessed 21 February 2010.
[53] Interview with Mr. Michael Swanson, Project Engineer, Daft-McCune-Walker, email interview, 2 April 2010.
[54] Dr. Mark D. Pritt, personal interview with the author, 17 February 2010.
[55] Interview with Jacob Pritt, Harvard student majoring in computer science, Skype interview, 19 February 2010.
[56] Jonathan S. Harbour, *Beginning Java Game Programming, Second Edition* (Boston: Thomson Course Technology PTR, 2008), 50.
[57] Interview with Brian Staab, Remote Sensing Scientist/Engineer, ITT Geospatial Systems, email interview, 29 March 2010.

coordinate "hyper-planes" in more dimensions than we can visualize and then analyzed algebraically.[58]

We remember Descartes today for his famous statement, "I think, therefore I am." His greatest work, however, was bringing algebra and geometry together with the development of analytic geometry. Because of this innovation, he is one of the most important mathematicians of all time. People all over the world use his innovation, often without knowing it, when they learn mathematics in school, when they navigate using GPS, and when they use computers. Along with mathematicians, scientists and engineers, they owe a great deal to the man who plotted the future of mathematics: René Descartes.

[58] Interview with Dr. Manabu Torii, Research Assistant Professor at the Imaging Science and Information Systems (ISIS) Center in Georgetown University Medical Center, email interview, 1 April 2010.

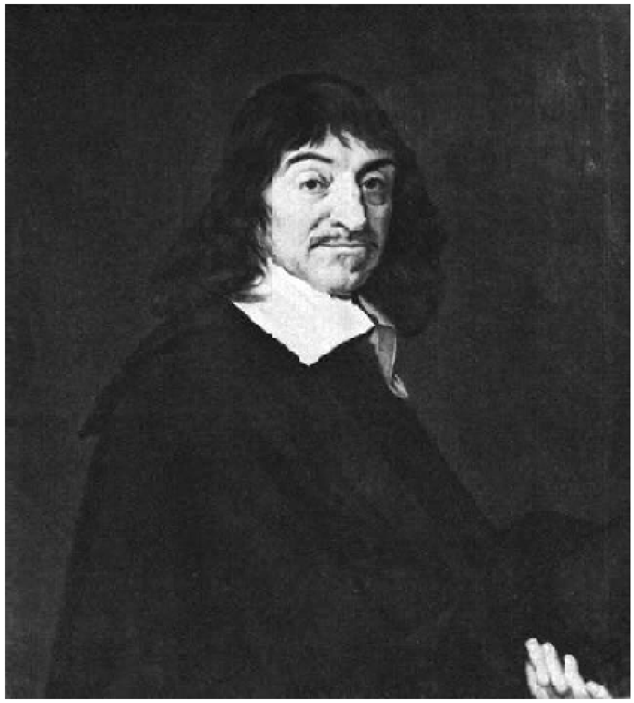
Appendix 1. Geometric diagram from Euclid's *Elements.*



This diagram from Euclid's *Elements* shows how complicated geometry eventually became under the Greeks, because they did not use analytic geometry.

Euclid, The Thirteen Books of Euclid's Elements, Vol. 11 of Great Books of the Western World, ed. Robert Hutchins, (Chicago: Britannica, 1989), 481.

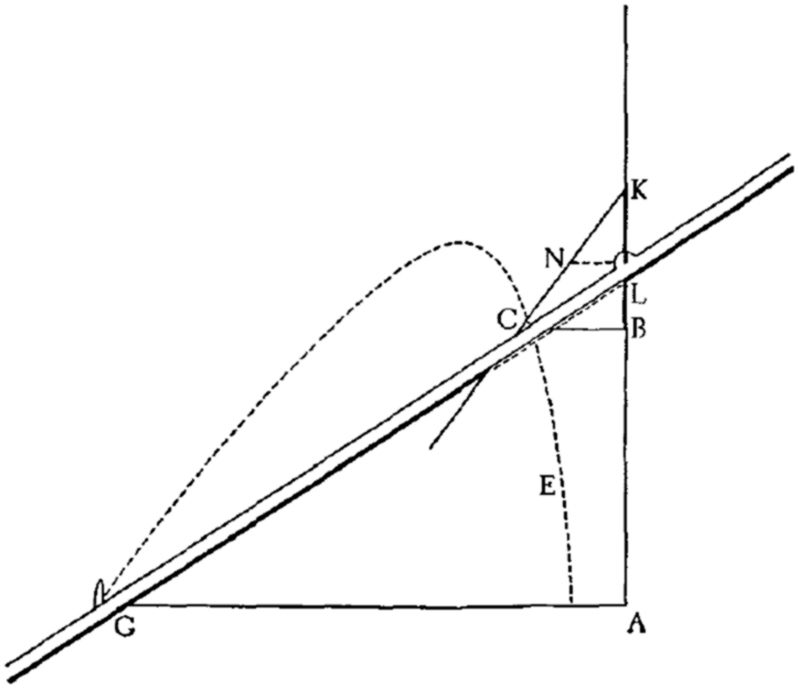
Appendix 2. Portrait of René Descartes



This portrait of René Descartes was painted by the Dutch artist Frans Hals in 1649.

Scientific-web.com, "René Descartes Images," available from http://www.scientific-web.com/en/Physics/Biographies/ReneDescartesImages.html, accessed 8 March 2010.

Appendix 3. Descartes' diagram of coordinate axes.



This diagram from Descartes' *Geometry* is the first example of Descartes using the Cartesian coordinate plane. GA is the *x* axis, AB is the *y* axis, and the point A is the origin.

René Descartes, *The Geometry,* Vol. 31 of *Great Books of the Western World,* ed. Robert Hutchins, (Chicago: Britannica, 1989), 306

Primary Sources

Books

Apollonius. *Conics I,* Vol. 11 of *Great Books of the Western World,* edited by Robert Hutchins. Chicago: Britannica, 1989.

In this book, Apollonius describes his work with conics. I studied this book because it helped pave the way towards analytic geometry. Apollonius was one of the first mathematicians to use coordinate axes. I discussed this book in the background section of my paper.

Descartes, René. *Discourse on Method,* Vol. 31 of *Great Books of the Western World,* edited by Robert Hutchins. Chicago: Britannica, 1989.

Descartes wrote *Discourse on Method* during the height of his philosophical career. I used it in my paper when I discussed Descartes' quote, "I think, therefore I am" and related it to his idea of analytic geometry by showing that both combine two distinct concepts to produce an entirely new idea.

Descartes, René. *The Geometry,* Vol. 31 of *Great Books of the Western World,* edited by Robert Hutchins. Chicago: Britannica, 1989.

Descartes' book, *The Geometry*, is the only book Descartes published about mathematics. Dr. Mark Pritt, Ph.D., helped me to understand the diagrams and proofs in this book. After a long search, and with help from Dr. Pritt, I found Descartes' use of coordinate axes on pp. 306-307. I quoted *The Geometry* multiple times in my paper.

Descartes, René. *The World or Treatise on Light*, translated by Michael S. Mahoney. Available from http://www.princeton.edu/~hos/mike/texts/descartes/world/worldfr.htm. Accessed 12 May 2010.

This is a book that Descartes wrote about physics. In chapter ten, I found a diagram of the solar system that shows the earth revolving about the sun. When Descartes found out that Galileo was condemned for this belief, he decided not to publish the book. I included this in my paper because it shows what could have happened to his book *Geometry*.

Euclid. *The Thirteen Books of Euclid's Elements*, Vol. 11 of *Great Books of the Western World,* edited by Robert Hutchins. Chicago: Britannica, 1989.

In Euclid's book *The Thirteen Books of Euclid's Elements,* I learned how Greek mathematicians approached geometry. Their diagrams eventually become too complicated and confusing without algebra. I used one of Euclid's diagrams to illustrate this in my appendix.

Pascal, Blaise. *Pensées*. Translated by W. F. Trotter. Charleston, SC: BiblioBazaar, LLC, 2009.

This book is a philosophical treatise. It was significant for my paper because of Pascal's criticism of Descartes, which I quoted in my paper. This criticism shows that although Descartes was well known, not everyone agreed with his ideas.

Letters

Descartes, René. Letter to Isaac Beeckman. 26 March 1619. In *The Philosophical Writings of Descartes.* Volume 3*: The Correspondence,* 1-3. Edited by John Cottingham, Robert Soothoff, Dugald Murdoch, and Anthony Kenny. New York: Cambridge University Press, 1991.

In this letter, Descartes writes to Beeckman about a discovery that he made for finding one's position from the stars. He asks him if anyone has discovered it yet. I used this letter in my paper as an example of the ideas that Descartes shared with Beeckman.

Descartes, René. Letter to Isaac Beeckman. 23 April 1619. In *The Philosophical Writings of Descartes.* Volume 3*: The Correspondence,* 3-4. Edited by John Cottingham, Robert Soothoff, Dugald Murdoch, and Anthony Kenny. New York: Cambridge University Press, 1991.

In this letter, Descartes informs his friend Beeckman that he started writing his book *Geometry*, and he thanks Beeckman for kindling his interest in mathematics. I used this letter to determine when Descartes started work on his book. I also cited it as evidence of the major influence that Beeckman had on Descartes.

Descartes, René. Letter to Isaac Beeckman. 29 April 1619. In *The Philosophical Writings of Descartes.* Volume 3*: The Correspondence,* 5. Edited by John Cottingham, Robert Soothoff, Dugald Murdoch, and Anthony Kenny. New York: Cambridge University Press, 1991.

In this letter, Descartes tells his friend Beeckman that he is leaving for Denmark. I cited it to show that Descartes traveled to Denmark.

Descartes, René. Letter to Isaac Beeckman. 17 October 1630. In *The Philosophical Writings of Descartes.* Volume 3*: The Correspondence,* 26-28. Edited by John Cottingham, Robert Soothoff, Dugald Murdoch, and Anthony Kenny. New York: Cambridge University Press, 1991.

In this letter, Descartes tries to persuade Beeckman that he did not plagiarize his ideas. I used this letter in my paper to show the conflict that developed between Descartes and his friend.

Descartes, René. Letter to Mersenne. November 1633. In *The Philosophical Writings of Descartes.* Volume 3*: The Correspondence,* 40-41. Edited by John Cottingham, Robert Soothoff, Dugald Murdoch, and Anthony Kenny. New York: Cambridge University Press, 1991.

In this letter to his friend Father Mersenne, Descartes explains why he decided not to publish his physics book *The World*. He wrote, “In fact I had intended to send you my *World* as a New Year gift [but] I took the trouble to inquire in Leiden and Amsterdam whether Galileo’s *World System* was available, for I thought I had heard that it was published in Italy last year. I was told that it had indeed been published but that all the copies had immediately been burnt at Rome, and that Galileo had been convicted and fined. I was so astonished at this that I almost decided to burn all my papers or at least to let no one see them.” I included this in my paper to show the fate that could have befallen his book *Geometry*.

Fermat, Pierre de. Letter to Mersenne. February, 1638. Available from <http://wlym.com/~animations/fermat/16380200%20Fermat%20to%20Mersenne.pdf>, accessed 20 February 2010.

In this letter, Fermat writes to Mersenne about Descartes and his progress in mathematics. I quoted it in my paper to show that many mathematicians were aware of Descartes’ work.

Leibniz, Gottfried Wilhelm. Letter to Philipp, 1679. In *The Philosophical Works of Leibniz*. Edited by George Martin Duncan. New Haven: Tuttle, Morehouse & Taylor, 1908.

In this letter, Leibniz, a Prussian mathematician, writes that reading Descartes' book was "very useful and very instructive." I quoted this letter in my paper to show that Descartes' work was well known.

Documents

Conduitt's account of Newton's life at Cambridge. Keynes Ms. 130.4, King's College Library, Cambridge. Accessed at http://www.newtonproject.sussex.ac.uk/view/texts/normalized/THEM00167.

This unfinished biography of Isaac Newton was written by John Conduitt, his nephew by marriage. I quoted it in my paper to show that Newton read Descartes' book *Geometry.*

Secondary Sources

Books

Boyer, Carl B. *History of Analytic Geometry*. Mineola, New York: Dover Publications, 2004.

This book describes the lives of many mathematicians and explains how their work contributed to analytic geometry. I used it in my paper to describe the works of the Greek mathematician Apollonius, the Persian mathematician Omar Khayyam, and the medieval mathematician Nicole Oresme.

Devlin, Keith J. *The Language of Mathematics: Making the Invisible Visible.* New York: Macmillan, 2000.

This book is about the history of mathematics and its application to technology. I used it in my paper because I found the story of the fly on the ceiling in it, which is a myth about how Descartes discovered analytic geometry. It gives no primary source for the story.

Gaukroger, Stephen. *Descartes: An Intellectual Biography.* New York: Oxford University Press, 1995.

This biography of Descartes includes several letters that he wrote to Princess Elizabeth of Bohemia. I quoted them in my paper to shed light on his childhood illnesses.

Giancoli, Douglas C. *Physics for Scientists & Engineers, 4th ed.* New Jersey: Pearson Prentice Hall, 2008.

This physics textbook uses analytic geometry and calculus, especially when working with vectors. I used it as an example of the use of analytic geometry in physics.

Glass, Julie. *The Fly on the Ceiling.* New York: Random House, 1998.

This is a beginning reader about Descartes' invention of analytic geometry. It is the book that got me interested in Descartes years ago. It is one example of many books that repeat the myth of the fly on the ceiling. As I noted in my paper, there is no primary source for this story.

Gray, Jeremy and Karen Hunger Parshal. *Episodes in the History of Modern Algebra (1800-1950).* Providence, RI: American Mathematics Society, 2007.

In this book, I learned about the Persian mathematicians who invented algebra, including Al-Khwarizmi and Omar Khayyam. I cited it when writing about the beginning of algebra.

Harbour, Jonathan S. *Beginning Java Game Programming, Second Edition.* Boston: Thomson Course Technology PTR, 2008.

In this computer programming book, I found several places where the author uses coordinate axes. It is an example of analytic geometry being used in computer programming. I quoted it in my paper as evidence of the usefulness of analytic geometry.

Hudson, John C. ed. *Goode's World Atlas 20th Edition.* Skokie, IL: Rand McNally, 2000.

This atlas explains the use of latitude, longitude, and coordinate systems in modern cartography. I used it when writing my paper because it showed me another application of analytic geometry to modern technology.

Kagan, Donald, Ozment, Steven, and Turner, Frank M. *The Western Heritage*. Upper Saddle River, New Jersey: Prentice Hall, 1998.

In this book, I learned about the Netherlands' war for independence from Spain. Descartes joined the Dutch army in 1618 under Maurice of Nassau, the son of William of Orange.

Nichols, Eugene D., Mervine L. Edwards, E. Henry Garland, Sylvia A. Hoffman, Albert Mamary, William F. Palmer. *Holt Algebra 1*. New York: Holt, Rinehart and Winston, 1986.

This is the mathematics textbook that I used to learn Algebra 1. It taught me about the Cartesian coordinate system. It proved a valuable source to me when writing my paper.

Nichols, Eugene D., Mervine L. Edwards, E. Henry Garland, Sylvia A. Hoffman, Albert Mamary, William F. Palmer. *Holt Algebra 2 with Trigonometry*. New York: Holt, Rinehart and Winston, 1986.

This is the mathematics textbook that I used to learn Algebra 2. It helped me understand conics. It also proved invaluable when writing my paper.

Reimer, Luetta and Wilbert Reimer, *Mathematicians are People, Too.* Palo Alto, California: Dale Seymour Publications, 1995.

This book portrays the lives of many mathematicians. When it presents Descartes' life, it recounts the story of the fly on the ceiling. This is another example of a secondary source that relates the myth without a primary reference.

Sasaki, Chikara. *Descartes's Mathematical Thought*. Volume 237 of Boston studies in the philosophy of science. New York: Springer Science+Business Media, 2003.

In this book, I learned about the Jesuit college Royal-Henry-Le-Grand at La Flèche. It was a famous school that was once a castle owned by Henri IV and later given to the Jesuits. Descartes went to this college from 1607 to 1615.

Vickers, K. M. and M. J. Tipler. *New National Curriculum Mathematics: Without Answers.* New National Curriculum Mathematics Series. Cheltenham, UK: Nelson Thornes, 1996.

In this mathematics book for teachers, I found another example of a secondary source that repeats the myth of how Descartes discovered analytic geometry after watching a fly on his ceiling. It gives no primary source for the story.

Watson, Richard. *Cogito, Ergo Sum: The Life of Rene Descartes*. Boston: David R. Godine, 2007.

In this biography of Descartes, I found a letter that he wrote to his friend Balzac about Italy. I cited it in my paper because it proves that he traveled to Italy.

Westfall, Richard S. *Never at Rest: A Biography of Isaac Newton.* New York: Cambridge University Press, 1983.

This biography describes Newton’s attitudes towards Descartes. I already knew that Newton had learned mathematics from Descartes’ *Geometry*, but I did not know that he later harshly criticized the book. I quoted this criticism in my paper.

Web Pages

National Space-Based Positioning, Navigation, and Timing Coordination Office. "Global Positioning System: Serving the World". Available from < http://www.gps.gov/>, accessed 21 February 2010.

From this webpage, I gained an understanding of how the Global Positioning System (GPS) uses analytic geometry. GPS relies on Cartesian coordinates to provide navigational information. I used this example in my paper to explain how the modern world uses analytic geometry.

Scientific-web.com. “René Descartes Images.” Available from http://www.scientific-web.com/en/Physics/Biographies/ReneDescartesImages.html, accessed 8 March 2010.

At this webpage, I found a portrait of Descartes painted by a Dutch artist in 1649. I included it in my appendix.

Smith, Kurt. “Descartes’ Life and Works”. *The Stanford Encyclopedia of Philosophy (Fall 2008 Edition)*. Edited by Edward N. Zalta. Available from <http://plato.stanford.edu/archives/fall2008/entries/descartes-works/>, accessed 19 February 2010.

This webpage contains biographical information on Descartes’ early life and travels. It also describes his friendship with Isaac Beeckman.

Interviews

Brown, Richard J, Ph.D., Director of Undergraduate Studies, Department of Mathematics, Johns Hopkins University. Email interview. 31 March 2010.

In this interview, I learned how topology, a form of geometry, depends on analytic geometry. Dr. Brown stated, “My field of research would not exist without analytic geometry.” This shows the impact that it has on modern mathematics.

Hill, Blake, Ph.D., Associate Professor of Biology and Chemistry, Johns Hopkins University. Email interview. 31 March 2010.

Dr. Hill explained how he uses analytic geometry in spectroscopy. Analytic geometry is also essential to solving the equations that arise from certain biological theories. This is an example of the impact of Descartes' innovation on modern science.

Mese, Chikako, Ph.D., Chair and Professor Department of Mathematics, Johns Hopkins University. Email interview. 31 March 2010.

In this interview, I learned that analytic geometry is used in differential geometry. I used this information in my paper.

Nicodemus, John, Teacher, Frederick Community College. Email interview. 29 March 2010.

Mr. Nicodemus teaches developmental algebra at Frederick Community College, and he answered several questions that I had about the importance of analytic geometry. I asked him if anyone else would have discovered analytic geometry if Descartes had not, and he replied that "…if Descartes hadn't been so bright to see how all those pieces fit together, someone named Pritt or Nicodemus would have done so eventually..."

Pritt, Jacob, Harvard student majoring in computer science. Skype interview. 21 February 2010.

Jacob goes to Harvard University. He explained to me how computer programmers use analytic geometry. I used this information in my paper.

Pritt, Mark D, Ph.D., Systems Engineer, Lockheed Martin Corp. Personal interview with the author. 17 February 2010.

This interview helped me to understand how modern mathematicians benefit from analytic geometry and how it is used in science and engineering today. Dr. Pritt spent time discussing Descartes' complex ideas with me, so I could understand and explain them in my paper.

Staab, Brian, Remote Sensing Scientist/Engineer, ITT Geospatial Systems. Email interview. 29 March 2010.

In this interview, I discovered how vital analytic geometry is to the field of image processing. For example, without it we would not have digital cameras. This is another example of how Descartes' innovation is used in our modern lives.

Stubbs, Christopher, Ph.D., Professor of Physics and of Astronomy, Harvard University. Email interview. 1 April 2010.

Dr. Stubbs explained to me how analytic geometry is used in astronomy to control telescopes with trigonometry. This is another example of the modern application of analytic geometry.

Swanson, Michael, Project Engineer with Daft-McCune-Walker. Email interview. 2 April 2010.

Mr. Swanson explained to me how engineers use analytic geometry for stormwater management, floodplain analysis, and road design for commercial buildings and housing developments. I used this information as examples of the application of analytic geometry to engineering.

Torii, Manabu, Ph.D., Research Assistant Professor, Imaging Science and Information Systems (ISIS) Center, Georgetown University Medical Center. Email interview. 1 April 2010.

In this interview, I learned how analytic geometry is used in text processing, which is important for classifying documents. I used this information in my paper to show how analytic geometry is used in our modern world.

Washington, Lawrence, Ph.D., Professor of Mathematics, University of Maryland. Email interview. 30 March 2010.

In this interview I discovered that analytic geometry is used in cryptography, the study of secret codes. Dr Washington wrote, “Analytic geometry was not important in number theory until around 1900, and it was not important in cryptography until around 1985. In both these fields, once analytic geometry started being used, people saw how useful it was and continued to make great use of it.” I cited this interesting fact in my paper.